Integrated Mathematics II

Standard 1
Algebra and Functions

CORE STANDARD

Linear and Absolute Value Equations and Inequalities

Solve systems of linear equations and inequalities in three variables by substitution and elimination. Solve problems that can be modeled using systems of linear equations. Solve equations and inequalities involving absolute value.

[Standard Indicators: IM2.1.4, IM2.1.5, IM2.1.7]

CORE STANDARD

Polynomials

Multiply polynomials, factor polynomials and divide a polynomial by a monomial.

[Standard Indicators: IM2.1.9, IM2.1.10]

IM2.1.1 Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.

Example: Given the function *f* below, find the indicated value, if possible.

- where *f* is represented by the set of ordered pairs {(3, 5), (2, -3), (1, 7), (0, 2)}, find the value of $f(1)$.
- where $f(x) = \sqrt{4 - x}$, find the value of $f(2)$ and $f(8)$.

IM2.1.2 Recognize and describe the relationships among the solutions of an equation, the zeros of a function, the *x*-intercepts of a graph and the factors of a polynomial expression.

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

IM2.1.3 Solve systems of linear equations and inequalities in three variables by substitution and elimination.

Example: Solve the system of equations: $x - 2y + 3z = 5$, $x + 3z = 11$, $5y - 6z = 9$.

IM2.1.4 Solve problems that can be modeled using systems of linear equations in three variables, interpret the solutions and determine whether the solutions are reasonable.

Example: Each week you can work no more than 20 total hours between the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

IM2.1.5 Graph piecewise-defined functions.

Example: Graph the function $f(x) = \begin{cases} x+2 & if\ x \le 0 \\ 3x-1 & if\ x > 0 \end{cases}$

IM2.1.6 Solve equations and inequalities involving the absolute value of a linear function.

Example: Solve the inequality $|x-5| \ge 8$ and graph the solution.

IM2.1.7 Use the laws of exponents for variables with exponents. Multiply, divide and find powers of variables with exponents.

Example: Simplify $a^2b^6(a^3)$, (n + 2) (n – 2), and $(n + 2)^2$.

IM2.1.8 Add, subtract and multiply polynomials and divide polynomials by monomials.

Example: Subtract (4x2 – 7x + 2) – (x2 + 4x – 5), multiply (n + 2)(4n – 5) and divide $4x^3y^2 + 8xy^4 – 6x^2y^5$ by $2xy^2$.

IM2.1.9 Factor common terms from polynomials and factor quadratic expressions.

Example: Factor $4ax + 3ay + 4bx + 3by$, $2x^2 – 7x + 3$ and $9a^2 – 4$.

Standard 2

Geometry and Measurement

CORE STANDARDS

Angles and Lines

Understand the relationship between special angles created by parallel lines and transversals.

[Standard Indicator: IM2.2.2]

CORE STANDARDS

Congruence and Similarity

Develop simple geometric proofs involving congruent and similar polygons.

[Standard Indicator: IM2.2.6]

CORE STANDARDS

Geometric Proof and Reasoning

Understand the differences among supporting evidence, counterexamples and actual proofs. Be able to develop simple geometric proofs, providing reasons for each statement, involving the following topics:

- parallel lines and transversals;
- congruent and similar polygons, particularly triangles; and
- circles.

[Standard Indicators: IM2.2.6, IM2.2.23]

CORE STANDARDS

Triangles

Prove the Pythagorean Theorem and its converse and use them to solve problems. Define trigonometric functions in terms of angles of right triangles and use them to solve problems.

[Standard Indicators: IM2.2.13, IM2.2.16, IM2.2.18]

CORE STANDARDS

Circles

Define, deduce and use formulas for and prove theorems for:

- radius, diameter, arc, chord, secant and tangent;
- measures of arcs and related angles (central, inscribed, and intersections of secants and tangents); and
- circumference, arc length, and areas of circles and sectors.

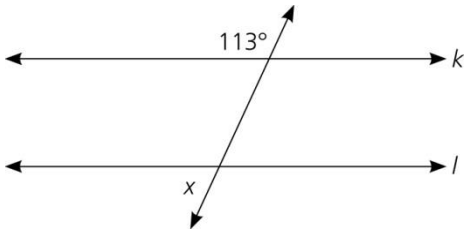
[Standard Indicators: IM2.2.20, IM2.2.21, IM2.2.22]

IM2.2.1 Construct congruent segments and angles, angle bisectors, perpendicular bisectors, and parallel and perpendicular lines by using appropriate geometric construction tools. Explain and justify the process used.

Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

IM2.2.2 Recognize, use and justify the relationships between special pairs of angles formed by parallel lines and transversals.

Example: In the diagram, the lines k and l are parallel. What is the measure of angle x? Explain your answer.



IM2.2.3 Identify and apply properties of and theorems about parallel and perpendicular lines, write equations of parallel and perpendicular lines, and develop simple geometric proofs involving parallel and perpendicular lines.

Example: Find an equation of a line perpendicular to $y = 4x - 2$ that contains the point (4, 1).

IM2.2.4 Identify, justify and apply properties of planes.

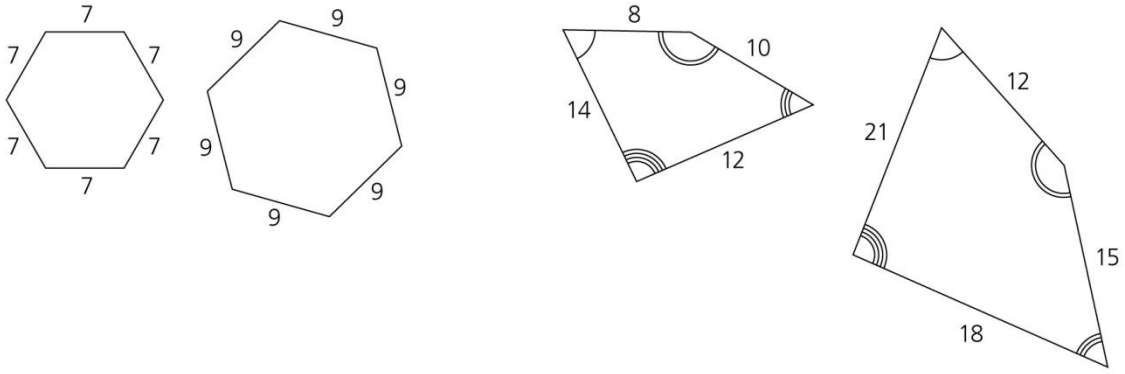
Example: Describe the intersection of plane R with parallel planes S and T.

IM2.2.5 Describe the intersection of two or more geometric figures in the plane.

Example: What is the maximum number of times two circles of the same size can intersect? Three circles? Six circles? Explain your reasoning.

IM2.2.6 Develop simple geometric proofs involving congruent and similar polygons and provide reasons for each statement.

Example: Prove that the following pairs of polygons are similar.



IM2.2.7 Describe, classify and recognize relationships among the quadrilaterals, such as squares, rectangles, rhombuses, parallelograms, trapezoids and kites.

Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

IM2.2.8 Prove and apply theorems about parallelograms and trapezoids (including isosceles trapezoids) involving their angles, sides and diagonals. Prove that the given quadrilaterals are parallelograms, rhombuses, rectangles, squares or trapezoids (as appropriate).

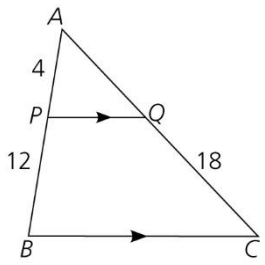
Example: Prove that both pairs of opposite sides of a parallelogram are congruent.

IM2.2.9 Define, identify, construct and solve problems involving perpendicular bisectors, angle bisectors, median and altitudes in triangles.

Example: Draw several triangles. Construct their angle bisectors. What do you notice?

IM2.2.10 Use theorems to show whether two triangles are congruent (i.e., SSS, SAS, ASA) or similar (i.e., AA, SAS, SSS).

Example: In the example below, prove that ΔABC and ΔAPQ are similar and use the similar triangles to compute the length of $\overline{AQ}$.

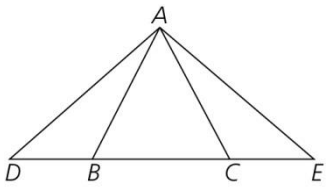


IM2.2.11 Prove and apply the triangle inequality theorem.

Example: Can 7, 15 and 5 be the sides of a triangle? Prove your answer is accurate.

IM2.2.12 Prove and apply the isosceles triangle theorem and its converse.

Example: Given isosceles ΔABC and DB ≅ EC, prove ΔDBA ≅ ΔECA.



IM2.2.13 Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane.

Example: Triangle *DEF* has vertices *D*(2, 4), *E*(0, 2), and *F*(3, -1). Determine whether Δ*DEF* is a right triangle.

IM2.2.14 Prove and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle.

Example: In Δ*ABC* with right angle at *C*, draw the altitude $\overline{CD}$ from *C* to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

IM2.2.15 Use special right triangles (e.g., 30°- 60° and 45° - 45°) to solve problems.

Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

IM2.2.16 Define and use the trigonometric functions sine, cosine and tangent in terms of angles of right triangles.

Example: In Δ*ABC*, tan *A* = $\frac{1}{5}$. Find sin *A* and cos *A*.

IM2.2.17 Deduce and apply the area formula $A = \frac{1}{2}ab\sin C$, where *a* and *b* are the lengths of two sides of a triangle and *C* is the measure of the included angle formed by the two sides.

Example: Find the area of an equilateral triangle with sides five units long.

IM2.2.18 Solve problems that can be modeled using right triangles, including problems that can be modeled using trigonometric functions. Interpret the solutions and determine whether the solutions are reasonable. Use technology as appropriate.

Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

IM2.2.19 Construct the circle that passes through three given points not on a line. Construct tangents to circles. Circumscribe and inscribe circles. Justify the process used.

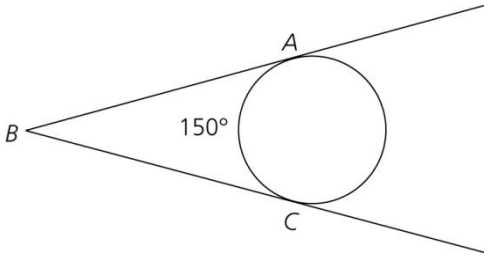
Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

IM2.2.20 Define, deduce and use formulas for, and prove theorems for radius, diameter, chord, secant and tangent.

Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

IM2.2.21 Define, deduce and use formulas for, and prove theorems for measures of arcs and related angles (i.e., central, inscribed and intersections of secants and tangents).

Example: Find the measure of $\angle ABC$ in the diagram below.



IM2.2.22 Define, deduce and use formulas for, and prove theorems for measures of circumference, arc length, and areas of circles and sectors.

Example: Use an appropriate theorem to find the sum of the interior angles of a convex n-gon.

IM2.2.23 Develop simple geometric proofs involving circles and provide reasons for each statement.

Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

IM2.2.24 Identify, justify and apply properties of prisms, regular pyramids, cylinders, right circular cones and spheres.

Example: Which of these properties of a cylinder is not true, and how do you know?

- The bases are congruent.

- The sections produced by the intersection of a cylinder and two parallel planes are congruent.
- The volume is the product of the area of the base and the altitude.
- The lateral area of a right circular cylinder is the product of the altitude and the base.

IM2.2.25 Solve problems involving congruent and similar solids.

Example: Explain how the surface area and volume of similar cylinders are related.

Standard 3
Data Analysis and Probability

CORE STANDARDS

Combinatorics and Probability

Use permutations, combinations and other counting methods to determine the number of ways that events can occur. Calculate the probability of compound events, and analyze probabilities to interpret odds and risks of events.

[Standard Indicators: IM2.3.6, IM2.3.7]

IM2.3.1 For bivariate measurement data, create a scatter plot; describe its shape; and determine regression coefficients, regression equations and correlation coefficients using technological tools.

Example: Measure the wrist and neck size of each person in your class and make a scatter plot. Interpret the correlation coefficient and the least squares regression line. Graph the residuals and evaluate the fit of the linear equation. Which line is a better fit? Explain your reasoning.

IM2.3.2 Display and analyze bivariate data where at least one variable is categorical.

Example: It is sometimes perceived that many retired people living in Florida live in mobile homes. Describe how you would gather, display and analyze the data to determine if more people 65 and older lived in mobile homes in Florida.

IM2.3.3 Recognize how linear transformations of univariate data affect shape, center and spread.

Example: Discuss whether you would use the mean or median to measure the center of each of the data below and why you made your particular choice.

- The yield of soybeans (bushels per acre) for a sample farm in Indiana.
- The prices of cars associated with each household in your neighborhood.

IM2.3.4 Calculate and interpret the correlation coefficient. Use the correlation coefficient and residuals to evaluate a “best-fit” line.

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

IM2.3.5 Construct sample spaces and probability distributions in simple cases and use them to solve problems.

Example: A couple plans to have children until they have a boy or until they have four children, whichever comes first. List the outcomes in the sample space for this experiment. What is the expected number of children for this couple?

IM2.3.6 Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risks of events.

Example: When a die is rolled three times, what is the probability of obtaining a 6, followed by any even number, followed by a 4?

IM2.3.7 Use permutations, combinations and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.

Example: There are five students who work in a bookshop. If the bookshop needs three people to operate, how many days straight could the bookstore operate without the same group of students working twice?

Standard 4

Discrete Mathematics

IM2.4.1 Use the properties of matrix addition, subtraction and scalar multiplication to solve problems.

Example: The table below shows the number of bound books produced during one shift at two publishing companies. Write a matrix to represent one day's total output at the two plants. Use your results to find the differences among production totals at the plants. Which plant produces more bound books with no graphics?

	Hardback With Graphic	Hardback No Graphic	Softback With Graphic	Softback No Graphic
Publisher A	500	800	950	1,900
Publisher B	600	700	1,000	1,600

IM2.4.2 Create matrices to organize and store data categorized by two variables and interpret the meaning of a particular entry in a matrix.

Example: Write a matrix C to represent the data in the table below. Find element c_{24}. What does this element represent?

The Types of Books Chosen by Mr. Smith's Language Arts Classes				
	Fiction	Nonfiction	Science Fiction	Sports
Boys	15	20	18	22
Girls	18	17	10	15

IM2.4.3 Use the properties of matrix multiplication, including identity and inverse matrices, to solve problems.

Example: Explain how two matrices can be multiplied and what the dimensions of the product matrix will be.

IM2.4.4 Represent a system of equations in two or three variables as a matrix equation $Ax = b$ and use technology to find $x = A^{-1}b$.

Example: Alana's Boutique is selling faux pearls for the following prices:

- 2 grey faux pearls and 3 black faux pearls cost $8.25.
- 3 grey faux pearls and 4 black faux pearls cost $11.25.

Let x = the cost of one grey pearl. Let y = the cost of one black pearl. Write the system as a matrix equation. Use technology to find the cost of one grey pearl and the cost of one black pearl.

IM2.4.5 Model and solve problems using matrices.

Example: To prepare for a dance, a school needs to rent 100 chairs, 4 large tables and 10 punch bowls. Rental prices were collected from two rental shops with the following matrix representing the two rental shops:

	R_1	R_2
Chairs	$2	$2.50
Tables	$20	$15
Bowls	$6	$4

Which rental shop, R_1 or R_2, has the lowest price for the group of items?

IM2.4.6 Use and interpret relational conjunctions (*and*, *or*, *not*), terms of causation (*if... then*) and equivalence (*if and only if*). Distinguish between the common uses of such terms in everyday language and their use in mathematics.

Example: Write "If today is Sunday, then we have school tomorrow" as a propositional statement by defining statements p and q and using symbols.

IM2.4.7 Use truth tables to determine the truth values of propositional statements.

Example: Find the truth table for $(p \rightarrow q) \rightarrow (q \rightarrow p)$.

IM2.4.8 Recognize syllogisms, tautologies, flawed reasoning and circular reasoning.

Example: Is the following reasoning valid? How do you know? "Today is Sunday, and we have school tomorrow. If today is not Sunday, then we have school tomorrow. Therefore, we do not have school tomorrow."

IM2.4.9 Construct and interpret directed and undirected graphs, decision trees, networks and flow charts.

Example: There are two islands in the River Seine in Paris. The city wants to construct four bridges that connect each island to each side of the riverbank and one bridge that connects the two islands directly. The city planners want to know if it is possible to start at one point, cross all five bridges, and end up at the same point without crossing a bridge twice. Use a graph to help solve this problem.

IM2.4.10 Use critical-path analysis to solve scheduling problems.

Example: Write a critical task list for redecorating your room. Some tasks depend on the completion of others and some may be carried out at any time. Use a graph to find the least amount of time needed to complete your project.

PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve

problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
    - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
    - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or not: Creating a high school diploma that counts.* Washington, DC: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston VA: author.